

The entire chromosphere is supposed to consist of innumerable small eruptions or jets of highly-heated gases similar to the so-called "metallic" prominences, which are only the more pronounced manifestations of the same eruptive agencies.

Evidence for this is found in the characteristic features of the chromosphere, and in the detailed structure of many of the Fraunhofer lines, which show wide emission lines underlying the narrow absorption lines. These ill-defined bright lines in the normal solar spectrum are distinctly displaced towards the violet, indicating a strong uprush of the hotter gases, whilst the narrow absorption lines are almost in their normal positions, and appear to indicate a slow and uniform descent of the absorbing gases.

The final conclusion is that the flash spectrum represents the emission of both ascending and descending gases, whilst the Fraunhofer spectrum represents the absorption of the descending gases only.

"On the Electrodynamic and Thermal Relations of Energy of Magnetisation." By J. LARMOR, M.A., D.Sc., Sec. R.S. Received January 2,—Read January 22, 1903.

1. There appears to be still some uncertainty as to the principles on which the energy of magnetised iron is to be estimated, and the extent to which that energy is electrodynamically effective. The following considerations are submitted as a contribution towards definite theoretical views.

The electrokinetic energy of a system of electric currents  $\iota_1, \iota_2, \dots$ , flowing in complete linear circuits in free aether, is known to be

$$\frac{1}{2}(\iota_1 N_1 + \iota_2 N_2 + \dots);$$

wherein  $N_1$  is the number of tubes of the magnetic force ( $\alpha, \beta, \gamma$ ) that thread the circuit  $\iota_1$ , and is thus equal to  $\int(l\alpha + m\beta + n\gamma)dS$  extended over any barrier surface  $S$  which blocks that circuit, ( $\alpha, \beta, \gamma$ ) being circuital (*i.e.*, a stream vector) so that all such barriers give the same result. As under steady circumstances ( $\alpha, \beta, \gamma$ ) is also derivable from a magnetic potential  $V$ , which has a cyclic constant  $4\pi\iota$  with regard to each current, this energy assumes the form

$$\frac{1}{8\pi} \sum \int V \left( l \frac{\partial V}{\partial x} + m \frac{\partial V}{\partial y} + n \frac{\partial V}{\partial z} \right) dS,$$

in which the integrals are now extended over both faces of each

barrier surface. This is equal by Green's theorem to the volume-integral

$$\frac{1}{8\pi} \int (\alpha^2 + \beta^2 + \gamma^2) d\tau$$

extended throughout all space. This latter integral is in fact taken in most forms of Maxwell's theory to represent the actual distribution, in all circumstances whether steady or not,\* of the electrokinetic energy among the elements of volume of the aether, in which it is supposed to reside as kinetic energy.

2. The most definite and consistent way to treat magnetism and its energy is to consider it as consisting in molecular electric currents; so that in magnetic media we have the ordinary finite currents, combined with molecular currents so numerous and irregularly orientated that we can only average them up into so much polarisation per unit volume of the space they occupy. So far in fact as the latter currents are concerned, the only energy that need or can occupy our attention is that connected with some regularity in their orientation, *i.e.*, with magnetisation, the remaining irregular part being classed with heat. If there were no such molecular currents, the magnetic force  $(\alpha, \beta, \gamma)$  in the aether would in steady fields be derived from a potential cyclic only with regard to the definite number of circuits of the ordinary currents. But when magnetism is present this potential is cyclic also with respect to the indefinitely great number of molecular circuits. The line integral of magnetic force round any circuit is  $4\pi(\Sigma\iota + \Sigma'\iota)$ , where  $\Sigma'\iota$  refers to the practically continuous distribution of magnetic molecular currents that the circuit threads. This latter vanishes when these currents are not orientated with some kind of regularity. If we extend the integral from a single line to an average across a filament or tube of uniform cross-section  $\delta S$ , with that line for axis, by multiplication by  $\delta S$ , we obtain readily the formula

$$\delta S \int (\alpha dx + \beta dy + \gamma dz) = \delta S 4\pi \Sigma\iota + 4\pi \int (A dx + B dy + C dz) \delta S$$

in which  $(A, B, C)\delta\tau$  represents the magnetisation in volume  $\delta\tau$ . Thus, after transposition of the last term, and removal of the factor  $\delta S$  after the average has now been taken, we obtain

$$\int \{(\alpha - 4\pi A) dx + (\beta - 4\pi B) dy + (\gamma - 4\pi C) dz\} = 4\pi \Sigma\iota$$

In other words this new vector  $(\alpha - 4\pi A, \beta - 4\pi B, \gamma - 4\pi C)$ , is derived from a potential which is cyclic in the usual manner with regard to the ordinary currents alone.

\* In the previous electric specification, the fictitious electric currents of aethereal displacement must be introduced when the state is not steady.

If we compare this result with the customary magnetic vectors of Kelvin and Maxwell, it appears that  $(\alpha, \beta, \gamma)$  must represent the "induction," and so will hereafter be denoted, after Maxwell, by  $(a, b, c)$ . The new vector, which has a potential cyclic with respect to the finite currents only, represents the "force," and will hereafter be denoted by  $(\alpha, \beta, \gamma)$ , whose significance is thus changed from henceforth. The "induction" on the other hand has not necessarily a potential, but is, by the constitution of the free aether, always circuital; that is, it satisfies the condition of streaming flow

$$\frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z} = 0.$$

The expression for the energy now includes terms

$$\frac{1}{2}(\iota_1 N_1 + \iota_2 N_2 + \dots)$$

for the ordinary currents  $\iota_1, \iota_2, \dots$ , where  $N_1, N_2, \dots$  are the fluxes, of *magnetic induction*, through their circuits; this transforms as usual into

$$\frac{1}{8\pi} \Sigma \int V(la + mb + nc) dS$$

over both faces of each barrier, which by Green's theorem is equal to

$$\frac{1}{8\pi} \int (a\alpha + b\beta + c\gamma) d\tau \dots \dots \dots \quad (i)$$

extended throughout all space. But there are also terms

$$\frac{1}{2}(\iota' N'_1 + \iota'_2 N'_2 + \dots)$$

for the molecular currents; now taking  $N'$  to be the cross-section of the circuit multiplied by the component of the averaged induction normal to its plane, and remembering that  $\iota'$  multiplied by this cross-section is the magnetic moment of this molecular current, it appears that  $\iota' N'$  is equal to the magnetic induction multiplied by the component of the magnetic moment in its direction, and therefore  $\frac{1}{2}\Sigma\iota' N'$  is equal to

$$\frac{1}{2} \int (Aa + Bb + Cc) d\tau.$$

Thus the magnetic circuits add to the energy the amount\*

$$\frac{1}{2} \int (A\alpha + B\beta + C\gamma) d\tau \dots \dots \dots \quad (ii)$$

together with

$$2\pi \int (A^2 + B^2 + C^2) d\tau \dots \dots \dots \quad (iii)$$

\* [These energies as here determined are *kinetic*; if they are (as is customary) to be considered as *potential*, their signs must be changed. Cf. 'Phil. Trans.', A, 1894, p. 806.]

The formula (i) is usually taken, after Maxwell's example, to represent the energy of the electrokinetic field. It here appears that it represents only the part of the energy that is concerned with the currents, arising from their mutual interactions and the interactions of the magnets with them; that there exists *in addition* a quantity (ii) which is that taken by Maxwell as the energy of magnetisation in the field ( $\alpha, \beta, \gamma$ ), and also a quantity (iii), which is purely local and constitutive, of the same general type as energy of crystallisation. The question arises whether (iii) is a part of the intrinsic energy of magnetisation of different kind from (ii), in that it cannot even partially emerge as mechanical work, or on the contrary the usual formula (ii) must be amended. See §§ 5, 8. In any case the dynamics of the field of currents (when there are no irreversible features) involves only that part of the energy function in which the currents operate, thus excluding both (ii) and (iii).

3. The simplest example is that of a coil of  $n$  turns carrying a current  $i$ , wound uniformly on a narrow iron ring-core, of cross-section  $S$  and length  $l$ . On the present basis the energy is made up of an electrodynamic part  $\frac{1}{2}nB^2S$  and a magnetic part  $\frac{1}{2}BS^2l$ ; as  $4\pi ni = Bl$  by the Amperean circuital law, these parts are

$$\frac{1}{8\pi} B^2v \text{ and } \frac{1}{2}BS^2l,$$

when  $v$  is the volume of the core; they make up *in all*  $B^2/8\pi$  per unit volume instead of the usual  $B^2/8\pi$ . The former part is mechanically available. The question has been raised by Lord Rayleigh\* whether the latter part, which includes the very large term (iii) above, namely  $2\pi l^2v$ , in the case of iron, has any considerable mechanical effectiveness; the question can only arise when it belongs in part to *permanent magnetism* whose ultimate annulment can induce a current,—when the current vanishes the energy of permanent magnetism, in the present case represented by (iii) alone, is the only part of the energy of the system that remains. The conclusion reached by him is that it cannot be annulled quick enough, when the ring carries a coil, to develop any considerable available electric energy by induction.

4. We may form a rough illustration of the mechanical rôle of this purely magnetic energy by considering, as the analogy of the currents, a branching system or network of pipes carrying liquids, in one of which a turbine is located, to be driven by the stream, which will be supposed to be an alternating one. The flow will be directed more fully into this particular pipe, and higher pressure will also be attained, after the manner of the hydraulic ram, if it communicates at the side with an expandible reservoir into which the liquid can readily

\* 'Phil. Mag.,' 1885: also 'Archives néerlandaises, vol. 2, 1891, p. 6, reprinted in 'Phil. Mag.,' 1902, and in 'Scientific Papers,' vol. 4, No. 272.'

force its way, to be expelled again by the elasticity of its walls when the stream begins to set in the reverse direction. This increase of kinetic pressure on the turbine roughly represents the electromotive pressure on a motor due to the increased magnetic flux, and the energy spent in expanding the reservoir as it fills up represents the energy of magnetisation of the iron. If things were perfectly reversible in the reservoir, that is if the iron were perfectly soft, the latter energy would rise and fall concomitantly with the alternations of pressure on the motor, but of course if its temperature remained constant it would contribute nothing to the energy driving the motor, which must be introduced into the system from an extraneous source. But if there are frictional resistances involved in filling the reservoir, the operations will not be perfectly reversible, and mechanical energy will be lost in it by conversion into heat; and moreover on account of the phase of its changes getting out of step—still more by permanent delays such as are classed under hysteresis—it will operate less efficiently in directing the stream of energy towards the turbine. Both these statements have analogical application to the iron in a magnetic circuit.

An example is provided by the ring-coil aforesaid. Suppose that when the current has ceased in the coil the core retains permanent magnetism, its energy being the latter term in the formula above. This corresponds to the reservoir becoming temporarily choked, so that it retains its contents after the pressure that drove the liquid into it has been removed. The question arises whether this retained energy is available for mechanical work. The present aspect of the matter appears to lead to the conclusion (Lord Rayleigh's) that it will not be available to any considerable extent unless its pressure in the reservoir is considerable, that is, in the magnetic case, unless the iron is not very receptive of magnetisation.

The paradox that energy of residual magnetism, which is outside the electrokinetic system, can on running down affect that system, shows that the circumstances are more general than an analogy of a pure dynamical system of finite number of degrees of freedom can illustrate. In fact the equations of dynamics imply permanent structure of the system; whereas in Professor Ewing's illustrative model of paramagnetisation, when the displacement is great enough the structure changes by the component magnets toppling over,\* and after the general disturbance thus set up has subsided with irrecoverable loss of energy into heat, there remains a new structure to deal with. The only way to estimate the available part that may be latent in the great store of energy of residual magnetism of an iron core is thus by the empirical process of detailed experiment. Lord Rayleigh has inferred

\* The effective susceptibility  $dI/dH$  becoming enormous in the steep part of the characteristic curve.

from the form of the curve of hysteresis for retentive iron in high fields that the fraction that is directly available at the actual temperature must always be small, and he supports the inference by considerations of the nature of the above analogy; in the absence of hysteresis there would be no such direct availability. He derives the practical result that a complete magnetic circuit is deleterious for induction coils in which length of spark is the *desideratum*, the increased total induction attained inside the ring-core being more than neutralised by the diminished promptness of magnetic reversal.

In fact, if the core, laminated so as to have merely negligible conductivity, is surrounded by a perfectly conducting coil or sheath, and its permanent magnetism is removed at constant rate  $-dI/dt$  by an ideal process applied to it, the intensity of induction in the core will diminish at the rate  $-4\pi dI/dt$ ; and this defect of induction must be made good by the influence of the current thereby induced in the sheath, as otherwise there would be a finite electromotive power in it, which is impossible on account of its perfect conductivity. This restored induction is of the form  $H' + 4\pi I'$ , where  $I'$  is the magnetism induced by the force  $H'$  due to the induced current; thus

$$\frac{d}{dt}(H' + 4\pi I') + 4\pi \frac{dI}{dt} = 0,$$

and the actual total rate of fall of magnetisation is diminished to  $\frac{dI}{dt} - \frac{dI'}{dt}$ , which is only the fraction  $\frac{1}{4}\pi \frac{dH' / dI'}{dt}$  of the constrained loss of retained magnetism  $dI/dt$ . In this most favourable case the action of the coil or sheath thus delays the time-rate of loss of permanent magnetism in the core in the ratio  $(4\pi\kappa')^{-1}$ , where  $\kappa'$  is the effective permeability for small additional force under the actual circumstances; that is, the delay in reversal more than compensates the gain in induction.

5. There remains another question, when viscous and other hysteretic effects are practically absent so that the changes of magnetisation exactly keep step with those of the currents, and the degree of availability of *residual* magnetic energy thus does not arise;—whether the energy of the magnetisation comes from the store of heat of the material and is thus concomitant with a cooling effect when no heat is supplied, or whether it is in part intrinsic inalienable energy of the individual molecules merely temporarily classed as magnetic. So far as it may be the latter, it must for each element of volume depend on the state of that element alone, like the part (iii) of § 2. It has already been seen that no part of (ii) or (iii) can be supplied from the electrodynamic field. This points to the intrinsic energy of paramagnetism, except an unknown fraction of the local part (iii), which depends only on the state of polarisation of the element of the medium, being

derived from purely thermal sources; and the following thermodynamic argument\* will strengthen this conclusion.

If the value of the magnetic susceptibility  $\kappa$  for any material is a function of the temperature, we can perform a Carnot reversible cycle by moving a small portion (say a sphere) of the substance in the permanent field of a system of magnets supposed held rigidly magnetised by constraints. We can move it into a stronger region  $H_2$  of this field, of varying strength  $H$ , maintaining it at the temperature  $\theta$  by a supply of heat from outside bodies at that temperature; we can then move it on further, having stopped the supply of heat, until its temperature becomes  $\theta - \delta\theta$ ; we can move it back again isothermally by aid of a sink of heat at this temperature until the stage  $H_1$  is reached, when further progress back adiabatically will restore it to its original condition. If  $\kappa$  is a function only of the strength of field and of the temperature, this cycle will be reversible. If  $E$  is the heat-energy supplied at temperature  $\theta$ , and  $W$  is the work done on the sphere by external bodies in the cycle, the principle of Carnot gives the relation

$$\frac{E}{\theta} = \frac{W}{\delta\theta}.$$

Now

$$W = - \frac{d}{d\theta} \left( \frac{1}{2} I_2 H_2 - \frac{1}{2} I_1 H_1 \right) \delta\theta$$

$$= - \frac{1}{2} \frac{d\kappa}{d\theta} (H_2^2 - H_1^2) \delta\theta, \text{ if } \kappa \text{ is small,} \dagger$$

when the cycle is taken such that the change of  $H$  along the adiabatic

\* This theoretical deduction of Curie's law has been already given substantially in 'Phil. Trans.,' A, vol. 190, 1897, p. 287.

The theory of diamagnetism, which assigns it to modification of conformation in the individual molecule by the inducing field rather than to average spacial orientation of the crowd of molecules, leads to a non-thermal origin as regards that part. The analogous question (*loc. cit.*) as to whether dielectric polarisation is mainly an affair of orientation of unaltered molecules like paramagnetism, or one of polarity due to internal deformation of the molecule like diamagnetism, is now answered by the experiments of J. Curie and Compan ('Comptes Rendus,' June 2, 1902). It appears that the dielectric coefficient of glass, for rapid changes, diminishes, but not very quickly, with fall of temperature, and that at temperatures below  $-70^\circ$  C. duration of charge ceases to have influence on its value. The electric excitation is thus analogous to diamagnetism and has no thermal bearing, its energy being self-contained in the molecule; the signs of the susceptibilities in the two cases are different, because the one is of static, the other of kinetic character. The sharpness of the Zeeman magneto-optic effect has already led ('Aether and Matter,' 1900, p. 351) in this direction, for it requires that the electric polarisation in the molecule shall be of isotropic type, so that there may be no axis of maximum susceptibility.

† This restriction is not necessary for the final result; if  $\kappa$  is not small,  $W$  and  $E$  have both to be multiplied by the same factor.

part of the path is negligible compound with that along the isothermal part. Thus

$$E = -\frac{1}{2}(H_2^2 - H_1^2)\theta \frac{d\kappa}{d\theta}.$$

Now the experiments of Curie on the relation of  $\kappa$  to  $\theta$  in weakly *paramagnetic* materials make  $\kappa$  vary inversely as  $\theta$ ; and this result has more recently been verified down to very low temperatures by Dewar and Fleming. This gives

$$E = \frac{1}{2}\kappa(H_2^2 - H_1^2).$$

Thus the movement of the magnetisable material at uniform temperature is accompanied by a supply to it of heat, equal to the mechanical work done by it owing to the attraction of the field; and this heat is just what is wanted to be transformed into the additional energy of intrinsic magnetisation (ii) of § 2. It is to be observed that in the actual experiments  $\kappa$  was small, and the other part (iii) of this energy therefore negligible: so that no conclusion as to the extent to which its source is thermal can be derived from Curie's law.

6. The uncertainties of § 4 do not of course affect the estimation of the loss of motive power arising from cyclic magnetic hysteresis, for we have here to do with the *mutual* energy of the applied field and the magnet, not the intrinsic local energy of the latter by itself. If the applied field is  $(\alpha, \beta, \gamma)$ , the total energy employed in polarising the magnetic molecules in volume  $\delta\tau$  is

$$(A\alpha + B\beta + C\gamma)\delta\tau.$$

So long as the polarisation is slowly effected against the resilience of reversible internal elastic forces this is stored as potential energy; but any want of reversibility involves degradation of some of it into heat, while if the field were instantaneously annihilated the molecules would swing back and vibrate, so that ultimately all would go into heat.

Let us pass the magnetic body through a cycle by moving it around a path in a permanent magnetic field  $(\alpha, \beta, \gamma)$ . An infinitesimal displacement of the volume  $\delta\tau$  from a place where the field is  $(\alpha, \beta, \gamma)$  to one where it is  $(\alpha + \delta\alpha, \beta + \delta\beta, \gamma + \delta\gamma)$  does mechanical work, arising from the magnetic attraction, of amount

$$(A\delta\alpha + B\delta\beta + C\delta\gamma)\delta\tau.$$

The integral of this throughout the whole connected system gives the virtual work for that displacement, from which the forces assisting it are derived as usual. Confining attention to the element  $\delta\tau$  the work supplied by it from the field, to outside systems which it drives, in traversing any path is thus

$$\delta\tau \int (Ad\alpha + Bd\beta + Cd\gamma),$$

the integral being taken along the path. If  $(A, B, C)$  is a function of  $(\alpha, \beta, \gamma)$ , that is if the magnetism is in part thoroughly permanent, and in part induced without hysteresis, so that the operation is reversible, this work must vanish for a complete cycle; otherwise energy would inevitably be created either in the direct path or else in the reversed one of the complete system of which  $\delta\tau$  is a part. Thus the negation of perpetual motion in that case demands that

$$Ad\alpha + Bd\beta + Cd\gamma = d\phi,$$

where  $\phi$  is a function of  $(\alpha, \beta, \gamma)$ , involving only even powers, and practically quadratic for small fields. Its coefficients are then the six magnetic constants for general aeolotropic material, no rotational quality in the magnetisation being thus allowable by the doctrine of energy. But if there is hysteresis, so that the cycle is not reversible,

$$-\delta\tau \int (Ad\alpha + Bd\beta + Cd\gamma),$$

or in vector product form  $-\delta\tau \int \mathbf{J} d\mathbf{B}$ , represents negative mechanical work done, or energy degraded, in the cycle.

In addition to this energy concerned with attraction, the external field expends energy in polarising or orientating the individual molecules against the internal forces of the medium, of aggregate amount

$$\delta\tau \int (\alpha dA + \beta dB + \gamma dC).$$

In any case, whatever the hysteresis, the sum of this second part and the first reversed is integrable independently of the path, giving

$$\delta\tau | A\alpha + B\beta + C\gamma |,$$

namely, the change in the total energy in the element, thus vanishing for a cycle which restores things to their original state, as it ought to do. The latter part is purely internal, and of merely thermal value as in § 5. The former part represents the averaged waste of direct mechanical energy in moving the iron armature through the cycle, and accounts for the heat thus evolved. It is the expression of Warburg and of Ewing for magneto-hysteretic waste of mechanical energy in driving electric engines; for a portion of a cycle it represents work partly degraded and partly stored magnetically.

7. Reverting to § 5, we may profitably illustrate by working out into detail a suggestion of Lord Rayleigh (*loc. cit.*). Consider a ring-coil of  $n$  turns with a flexible open core of soft iron of length  $l$  and cross-section  $S$ , whose flat ends are bent round until they face each other at a distance small compared with the diameter of section. We can apply Hopkinson's theory of the open magnetic circuit to trace the

transformation of the energy of attraction between these poles of the core, into electrokinetic energy of the coil, as the poles close up together. As frictional waste is not essential to this question, we can consider the coil to be a perfect conductor which will store all the energy without loss. We need not postulate that the iron is of constant permeability or devoid of hysteresis. When the distance between the pole-faces is  $x$ , let the current in the coil be  $\iota$ . The total energy is

$$\frac{1}{2}n\iota N + \text{energy of magnetisation};$$

and part of the latter may remain sub-permanent when  $\iota$  vanishes. The principle of the magnetic circuit gives, as  $\mu S = N$ , the formula

$$\frac{N}{\mu S} l + \frac{N}{S} x = 4\pi n\iota,$$

assuming as usual that the lines of magnetic force are conveyed straight across the air-gap between the pole-faces. Thus the electromagnetic energy  $T$ , equal to  $\frac{1}{2}n\iota N$ , is

$$2\pi \frac{n^2 \iota^2 S}{x + l/\mu};$$

and when  $x$  is diminished by  $-\delta x$ , its increment is approximately

$$-2\pi \frac{n^2 \iota^2 S}{(l/\mu)^2} \delta x + 4\pi \frac{n^2 \iota S}{l/\mu} \delta \iota \dots \quad (\text{i})$$

In this displacement the work expended from the electrodynamic system in mechanical attraction between the poles magnetised to intensity  $I$ , equal to  $\kappa N/\mu$ , is  $-IS2\pi I\delta x$ , which is

$$-\left(1 - \frac{1}{\mu}\right)^2 2\pi \frac{n^2 \iota^2 S}{(l/\mu)^2} \delta x \dots \quad (\text{ii})$$

Comparing (ii) with (i) it appears that in cases for which  $\mu$  is great, to which alone the principle of the magnetic circuit can be applied, the work of mechanical attraction by which the pole-faces can transfer potential energy to a spring placed between them, by compressing it, is concomitant with equal increase of the electrokinetic energy if the current do not change. As there is no source of energy, the current must therefore vary, and so that the total change of electrokinetic energy given by (i) and (ii) vanishes; that is, it must diminish by  $-\delta \iota$ , given by  $\delta \iota = \frac{1}{2} \frac{\mu \iota}{l} (1 + (1 - \frac{1}{\mu})^2) \delta x$ , which is practically equivalent to

$$\delta \iota = \frac{\mu \iota}{l} \delta x.$$

This result may be immediately verified by the Lagrangian process. As there is infinitely small resistance, the electric pressure  $\frac{d}{dt} \frac{\partial T}{\partial \iota}$  in

the coil must be infinitely small ; hence  $\iota \propto x + l/\mu$ , so that  $\frac{\delta\iota}{\delta x} = \frac{\mu}{l}$  when  $x$  is negligible.

The energy of magnetisation of the core, which is  $\frac{1}{2}\mathfrak{I}\mathcal{B}\mathcal{S}$ , where  $\mathfrak{I} = \kappa\mathfrak{B}/\mu$ , and therefore is  $\frac{1}{2}\frac{\kappa}{\mu}\left(\frac{4\pi n\iota}{x+l/\mu}\right)^2\mathcal{B}\mathcal{S}$ , is not included in this conservation. Its increase for a change  $\delta x$  is  $-\frac{\mu-1}{4\pi}\left(\frac{4\pi n\iota}{l/\mu}\right)^2\mathcal{S}\delta x$ , which is large compared with the quantities above. The fraction  $\mu^{-1}$  of it, which is comparable with the other variations, is compensated *thermally*, by absorption of the heat of the system, and has, therefore, only the limited availability of thermal energy. The remainder belongs intrinsically to the magnetisation, constituting mutual energy of contiguous molecules ; how much of it, as above expressed, is of thermal origin remains undetermined in the absence of calorimetric experiment.

8. The main points that it has been sought to bring out are as follows :—

(i) In an electrodynamic field there exists the usual specification of electrokinetic energy, but also *in addition* the energy of magnetisation of magnetic material.

(ii) This energy of magnetisation appears as made up of a part given by the ordinary formula, which (when paramagnetic) is derived from thermal sources, and so in the absence of hysteresis has the limited mechanical availability of thermal energy ; together with a local part which is to some extent thus available, but is also in part permanent intrinsic energy of the molecules, regarded temporarily as magnetic energy.

(iii) The law of Curie, that the susceptibility of weak paramagnetic substances is inversely proportional to the absolute temperature, is involved in these statements.

(iv) The extent of the direct (non-thermal) availability of *retained* magnetism can be inferred only by empirical procedure, for example, in general features by inspection of the hysteresis diagram, as pointed out by Lord Rayleigh.

